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ABSTRACT

This program description contains goals and objectives for five mathematics curriculum levels: three- and four-year-olds, five-year-olds, six- and seven-year-olds, eight- and nine-year-olds, and ten- and eleven-year-olds. A mathematics program overview for Smith College Campus School and a discussion of considerations for mathematics curriculum decision making are also presented. (MK)

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SMITH COLLEGE CAMPUS SCHOOL

MATHEMATICS PROGRAM

ED178302

SMITH COLLEGE CAMPUS SCHOOL
GILL HALL
NORTHAMPTON, MASSACHUSETTS 01063

SE 028 852

mathematics program

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1979

. . . it is true of arithmetic as it is of poetry that in some place and at some time it ought to be a good to be appreciated on its own account--just as an enjoyable experience, in short. If it is not, then when the time and place come for it to be used as a means or instrumentality, it will be just in that much handicapped. Never having been realized or appreciated for itself, one will miss something of its capacity as a resource for other ends. . .

(John Dewey 1930, p. 6)

. . . to understand something is to sense the simpler structure that underlies a range of instances, and this is notably true in mathematics.

(Jerome Bruner 1970, p. 106)

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PREFACE

PREFACE

This documentation of the Campus School's Mathematics Program is the outgrowth and culmination of several years work at the Campus School.

In the 1975-1976 school year, a description of past and current practice was prepared as part of the School Evaluation Report for the accreditation review by the Independent Schools Association of Massachusetts (I.S.A.M.). A math curriculum committee comprised of teachers, administrator, and parents was established which began by stating school-wide goals, identifying strands for the organization of content, and developing a series of objectives for several of the strands. This work was continued over the summer by a few teachers.

In the fall of 1976-1977, the school published a tentative curriculum document which stated the goals of the program and included the skill continuums developed by the committee. A re-formed committee continued throughout the year and examined several curriculum-related issues and problems, such as standardized tests, use and storage of materials, textbooks, development of additional strands.

During the present year, 1978-1979, the curriculum has been further refined and defined. An enlarged document was drafted by the Math Consultant and submitted to teachers, parents, administrators, and a member of the Smith College Department of Education and Child Study for review and response. This document is a revision, based on the recommendations of the reviewers. Plans have been made for the further development and implementation of the curriculum. The process is summarized in figure 1.

The program is designed specifically for the Smith College Campus School, taking into account the nature of its student population, the organization of the school into multi-age levels, and the school's position that curriculum and associated instruction are related both to content and to the characteristics of the learners as individuals and as members of groups.

This volume sets out the school goals in mathematics, the premises on which it is based, an overview of the school program, and for each level, an overview and statement of goals and objectives. It is expected that this volume will be supplemented by additional volumes for each level which outline the instructional plans to achieve the goals and objectives stated here. These supplementary volumes will be in loose-leaf format to facilitate the process of on-going development and frequent modification which is expected.

Paul M. Williamson
Math Consultant
March 1979

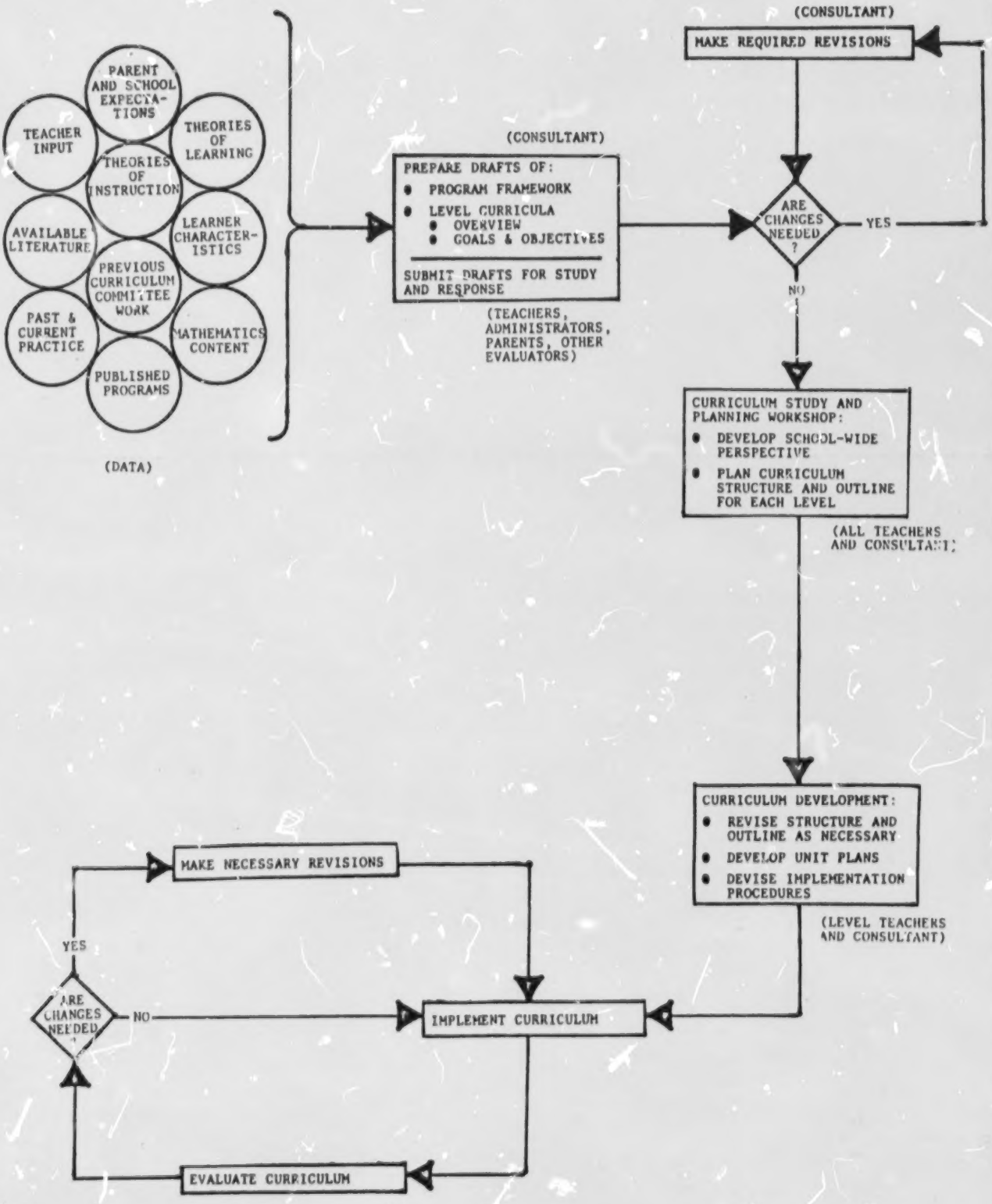

Fig. 1. CURRICULUM DEVELOPMENT IN MATHEMATICS 1978-1979

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I. SCHOOL GOALS IN MATHEMATICS

SCHOOL GOALS IN MATHEMATICS*

The mathematics program of the Campus School is intended to enable children to:

(1) represent mathematical concepts and procedures with models, and use these as an aid in problem solving.

(2) recognize and understand some of the unifying concepts and principles of mathematics, such as place value, equality, properties of operations.

(3) perform whole number computations using standard algorithms with reasonable speed and accuracy.

(4) understand the concept of fractional number and perform appropriate computations with fractional numbers, in common fraction or decimal form.

(5) recognize and appreciate the wide application of mathematical ideas and skills, both in school and daily life.

(6) apply mathematical concepts and skills to real-life situations, including (but not limited to) metric and customary measurement, money, and time.

(7) analyze and solve mathematical problems using systematic techniques, including estimating and judging the reasonableness of solutions.

(8) organize and interpret numerical data using tables, charts, graphs and simple statistical techniques.

(9) use geometric concepts and vocabulary to describe properties and relationships of shapes, both abstract and in the environment.

(10) solve simple geometric problems using informal methods and construct simple geometric figures.

(11) identify personal progress in mathematics and develop confidence as a learner of mathematics.

*Revised. Based on goals stated in Spring 1976 by the Math Curriculum Committee, composed of teachers, administrators, and parents.

PREMISES

The Campus School holds the position that the curriculum and associated instruction are related both to content and to the characteristics of the learners as individuals and as members of groups. In accord with this position, there are three premises on which the mathematics program is based.

Premise 1: The mathematics curriculum should be qualitatively different for each level of the school.

Corollary: The curriculum content and organization for each level should be that most appropriate to the learner.

Over the course of the Campus School years, significant changes take place in the cognitive, social, emotional, and physical development of children. These changes have profound implications for the basic questions of curriculum and instruction:

- What is to be learned?
- How is it to be learned/taught?

The premise specifically rejects the notion that each year's curriculum is more of the same, only harder. The content and ". . . structure must always be related to the status and gifts of the learner. Viewed in this way, the optimal structure of a body of knowledge is not absolute but relative." (Bruner 1966, p. 41) What is appropriate for the 6- or 7-year old is not appropriate for the 10- or 11-year old and so on. This is not to say that there are no broad, common aims for the total program of the school, or that there is no unifying pattern of development. It does say, however, that the curriculum for each level should have a particular emphasis, a particular set of goals, and make a particular contribution toward achieving the goals of the total program.

Premise 2: At all levels, the instructional practices used should be those most compatible with the nature of the learning task.

There are different kinds of learnings called for in the mathematics program--concepts, generalizations, procedures, number facts, attitudes, appreciations--and these require different types of learning experiences:

> The planning for effective attainment of a wide range of objectives requires several things. One of these is an awareness that the different behaviors involved in different areas of objectives require different types of learning experiences to attain them.
>
> (Taba 1962, p. 279)

At times, highly-structured, sequential experiences and practice will be required. At other times, exploration and guided experimentation will be in order. Neither alone will be sufficient. An experiential base is necessary, but mere experience is inadequate. This problem is not unique to mathematics learning. David Hawkins has addressed the same problem in

science curriculum, particularly with respect to ESS science, which was often interpreted and implemented as just "messing about." Hawkins asserts that such "messing about" must also be accompanied at the appropriate time by guided investigation and search for relationships (theorizing):

> Theorizing in a creative sense needs the content of experience and the logic of experimentation to support it. But these do not automatically lead to conscious abstract thought . . .. What we criticize for being too much too early, we must work to readmit in its proper place.
>
> (1971, p. 69)

Premise 3: The curriculum should provide both the stability and guidance of a defined educational plan and the opportunity for flexibility in its implementation.

There is a need to facilitate and assure continuity and direction for the child's progress toward achieving the goals of the mathematics program. It is also necessary to facilitate and ease the job of the teacher by optimizing the base of common experiences and competencies which children develop as they move through the school. This is particularly important in an organizational structure where children are regrouped frequently, moving from level to level and from classroom to classroom within levels.

It is assumed that the effectiveness of the total program can be increased by clearly specifying the institutional goals and objectives, and by defining the nature of the program components (level curricula) and the contribution of each to the total program. The first results in clarity of purpose and the second in articulation between components.

This premise recognizes that curriculum planning and development is a continuous, evolutionary process, and that the teacher must exercise his/her professional responsibility in curriculum decision-making. Flexibility and adaptability are needed to capitalize on the strengths, interests, and creative talents of both teachers and students, and to encourage their growth and development.

A dynamic tension exists between the need for continuity and direction on the one hand and the need for flexibility and adaptability on the other. To strike a balance between the two is never an easy matter. This curriculum is quite specific about the goals and objectives, but provides considerable lattitude to teachers in planning ways to accomplish them.

PROGRAM OVERVIEW

The school mathematics program as a whole should enable children to achieve the school goals in mathematics. Within the total program, however, the curriculum for each level has a particular emphasis and makes a particular contribution toward enabling the child to achieve those goals. Thus, the curriculums of the various levels are qualitatively different, and the developmental characteristics of children are an important determinant in this design.

In general, the program may be seen as developing from the concepts of number and space themselves toward the unifying ideas of the number and mathematical system. These broader understandings serve to simplify, organize, and therefore counter-balance a rapidly expanding base of mathematical knowledge and skill.

This trend of development is aptly described with a phrase used by David Hawkins (1971, p. 69): "from the concrete perceptual to the abstract conceptual."

Another way to characterize this development is to think of it as moving from a study of number in the world toward the study of the world of numbers.

Thus, the curricula will differ according to:

- the selection and organization of content and learning experiences;
- the relative emphasis placed on the development of concepts, facts, procedures, and generalizations;
- the extent to which the mathematical ideas are specific or general (generality of the conceptual framework);
- the extent to which mathematical ideas are differentiated and studied for their own sake (as well as in the integrated context of the child's broader experiences).

The way in which the program develops from level to level is summarized very briefly below. Much greater detail is provided in the goals and objectives of the level curriculums (see section V).

CAMPUS SCHOOL MATHEMATICS PROGRAM

SUMMARY

3- and 4-Year Olds

The curriculum at this level emphasizes background experiences and explorations with materials and their properties from which to develop concepts of number, space, and measurement. Emphasis is also placed on activities which stimulate the development of logical/mathematical reasoning.

5-Year Olds

The curriculum at this level emphasizes developing an understanding of number and the recognition and meaning of the numerals 1...20. Emphasis is also placed on experiences with measurement and geometry which extend the child's ability to discriminate and describe properties of objects and space and to order and classify logically on the basis of these properties.

6- and 7-Year Olds

The curriculum at this level emphasizes the development of the concepts of number and space themselves, and of the models, language, and symbols to express these ideas. Addition and multiplication concepts, facts, and skills are developed. Subtraction concepts and facts are introduced. Logical classification--a consideration of how things are the same or different--is an important correlate of number work. Measurement skills (length) are introduced and the concepts of volume and area are explored.

8- and 9-Year Olds

The curriculum at this level emphasizes skills with operations on numbers and the development of remaining number facts, particularly those for subtraction and multiplication. For number facts, the focus is on accurate recall of most facts and on strategies for quick solutions to those not recalled. Procedures for whole number computation are developed and extended to include standard algorithms for multiplication and division (by a 1-digit factor). Estimating and judging the reasonableness of answers are stressed, both in computation and measurement. Numeration is extended to reading, writing, and modeling larger numbers; fractional numbers are introduced. Concepts and skills are carefully developed and applied to real-world situations.

10- and 11-Year Olds

The curriculum at this level emphasizes the extension and mastery of whole number computation with speed and accuracy and the development of concepts and skills with fractional numbers. Particular attention is focused on mathematical/logical reasoning and on understanding and applying generalizations about numbers and the way they work. When appropriate, students are introduced to advanced topics, such as ratio and proportion, probability, statistical techniques, etc.

CONTENT OF THE MATHEMATICS PROGRAM

For purposes of analyzing and summarizing the content of the mathematics program, the following strands (subdivisions) are being used:

STRANDS*

LOGICAL/MATHEMATICAL REASONING
NUMBERS AND NUMERATION: WHOLE NUMBERS
OPERATIONS AND PROPERTIES: WHOLE NUMBERS
NUMBERS AND NUMERATION: FRACTIONAL NUMBERS
OPERATIONS AND PROPERTIES: FRACTIONAL NUMBERS
STATISTICS
(Includes: Graphing)
RELATIONS AND FUNCTIONS
(Includes: Ratio and Proportion, Percent, Probability)
GEOMETRY
MEASUREMENT
(Includes: Time, Money)

The placement and development of content within the program is summarized in Table 1, Analysis of Content. This table is provided only for convenience in analyzing and summarizing the content of the program. It is not intended to suggest or define an organization of the content for teaching. For instruction, the content must be "structured so that it can be most readily grasped by the learner" (Bruner 1966, p. 41). It is therefore expected that the organization of content for teaching will differ level by level.

---

*Modified slightly from the proposal of the Math Curriculum Committee, Spring 1976.

Table 1. ANALYSIS OF CONTENT

*	3 AND 4 YEAR OLDS	5 YEAR OLDS	6 AND 7 YEAR OLDS	8 AND 9 YEAR OLDS	10 AND 11 YEAR OLDS
LOGICAL/MATHEMATICAL REASONING	*Sorting by likenesses *Ordering by physical properties	*Classifying by 1 attribute *Ordering by single attribute *Ordering sets by number *Matching objects in series *Patterns: colors, shapes,..	*Classifying by 2 attributes ----------> (CONT) *Patterns with objects or numbers	*Intersecting sets, matrices, sub-sets. ----------> (CONT) ----------> (CONT)	*Statements: true/false, negation, inclusion/exclusion *Logical necessity *Charts & diagrams in problem solving
NUMBERS AND NUMERATION WHOLE NUMBERS	*Counting, up to 10 objects *Numerals: 1...5 *Comparing sets by matching	*Count, up to 20 objects *Rote counting to 100 *Comparing sets by number *Writing numerals 1...20 *Ordinals, first...fifth *Forming digits: 0...9	*Numbers to 999: reading, writing, modeling *Comparing numbers to 99 *Counting by 2,5,10 *Ordinals, first...tenth *Patterns of numbers *Symbols: +,-,=,>,<	*Numbers to 999,999: reading, writing, modeling *Place and total values of digits *Expanded notation (tot. val.) *Rounding: largest place val. *Number properties: odd, even, square, etc. *Factors, multiples----------> (CONT)	*Numbers through billions: reading, writing *Exponents *Place value system *Rounding to given place *Roman numerals (comparison)
OPERATIONS AND PROPERTIES: WHOLE NUMBERS — ADDITION		*Sums to 10: models	*Meaning and models *Recalling and solving basic facts *Estimating and adding: 2 digits w/regrouping 3 digits no regrouping *Recognizing problems which require addition----------> (CONT)	*Recalling basic facts with speed and accuracy----------> (CONT) *Estimating and adding any 2 numbers.----------> (CONT) *Column addition: 2-digit numbers----------> (CONT)	*Properties of addition
OPERATIONS AND PROPERTIES: WHOLE NUMBERS — SUBTRACTION		*Sums to 10: models	*Meaning and models *Recalling and solving basic facts (50% of combinations) *Modeling subtraction with regrouping *Recognizing problems which require subtraction----------> (CONT)	*Recalling basic facts with speed and accuracy----------> (CONT) *Estimating and subtracting: any 2 numbers----------> (CONT) *Solve for missing addend	*Properties of subtraction
OPERATIONS AND PROPERTIES: WHOLE NUMBERS — MULTIPLICATION			*Meaning and models----------> (CONT) *Recalling or solving basic facts: 2,5,10 *Modeling 1 digit x 2 dig. *Recognizing problems which require multiplication----------> (CONT) *Relating multiplication to addition----------> (CONT)	*Recalling or solving all basic facts. *Modeling multiplication: larger numbers *Estimating and multiplying: 1-dig. multiplier	*Recalling basic facts with speed and accuracy *Estimating and multiplying: 2- or 3-digit multiplier *Properties of multiplication
OPERATIONS AND PROPERTIES: WHOLE NUMBERS — DIVISION			*Readiness: informal sharing	*Meaning and models *Modeling division by 1-digit factor *Estimating and dividing: 1-digit divisor *Recognize problems which require division	*Estimating and dividing: 2-digit divisors. *Properties of division

Table 1. Analysis of Content (continued)

NUMBER & NUMERATION: FRACTIONS		*Readiness: Informal naming: halves	*Readiness: Informal naming: 1/2, 1/3, 1/4	Introduction *Naming fractions from models *Naming equivalent fractions from models *Decimal notation, to .1	*Naming fractions *Naming equivalent fractions *Decimal notation, to .001 *Applying identify property in renaming
OPERATIONS AND PROPERTIES: FRACTIONS				Introduction *Adding/subtracting like fractions, given models *Adding/subtracting like decimal fractions.	*Adding/subtracting: fractions, mixed numerals, unlike decimals *Multiplying with fractions and decimal fractions *Dividing with fractions, decimal fractions *Recognizing and applying properties of operations
RELATIONS AND FUNCTIONS		*Making lists *Recording data with tallies *Pictographs *Block graphs	----------> (CONT) *Charts & tables *Bar graphs ----------> (CONT)	*Line graphs ----------> (CONT) *Frequency tables ----------> (CONT) Range ----------> (CONT) *Ordered number pairs *Scales: maps & drawings	*Sampling to make estimations *Average, median *Ratios *Proportions *Percent *Probability
GEOMETRY	*Identifying common shapes: circle, square, etc. ---------- *Describing properties of shapes: curved, straight, etc. *Patterns: shape, size, position	----------(CONT) ----------(CONT) *Common solids: ball, can, box, cube	*Meaning of surface area *Estimating and measuring surface by counting repeated units *Tessellations *Meaning of capacity (vol.)	*Symmetry *Congruence (by superimposition) *Transformations: flips, slides, rotations *Perimeter *Area, constant area	*Properties of plane & solid figures *Angles *Compass and straightedge constructions *Constructing solid figures
MEASUREMENT		Time *Days: morning, afternoon, tomorrow, yesterday Days of the week	Time *Read clock: hour, half-hour *Read digital clock *Calendar units: days, weeks, months, year. *Read calendar	Time *Read clock: nearest minute *Relate time units: days/year weeks/year, hours/day, etc.	*Computing with measurements *History of measurement *Metric system *Selecting suitable metric units and measuring: length, volume, weight, temperature *Computing area and volume
		Money *Identifying coins	Money *Values of coins *Counting change, to $1	Money *Making change, to $10.00 *Computing with money.	
	Length *Comparing length: longer, shorter, etc. *Comparing height: taller, shorter, etc.	Length *Comparing length, directly or indirectly (string, etc.) *Ordering by length, up to 10 objects	Length *Measure length: m, cm	Length *Metric units: cm, dm, m *Estimating and measuring length to given precision *Expressing fractional measurements, e.g. 3.5 dm *Customary units	
	Weight *Comparing weight: heavier, lighter, etc. ----------> (CONT)	Weight ----------> (CONT)	Weight ----------> (CONT) *Ordering by weight ----------> (CONT)	Weight *Measuring weight: kg ----------> (CONT)	
	Volume *Comparing contents: more, less, empty, full, etc.	Volume *Comparing volume: holds more than, less than, same amount as	Volume *Measuring volume: non-standard units or liter *Ordering by volume ----------> (CONT)	Volume *Estimating and measuring volume: l, ml ----------> (CONT) *Customary units	
				Temperature *Measuring temperature: °C	

CURRICULUM DECISION-MAKING: SOME CONSIDERATIONS

Making a curriculum involves making decisions. What should be the content of the curriculum, and how should it be organized? What goals and objectives should be set for students? What experiences should be provided for students? Questions like these must be asked and answered. How are these matters to be decided?

Curriculum decisions are seldom simple or easy. They involve consideration of several complex and related factors. Among these are an understanding of the subject matter and its importance; the needs, abilities, and interests of the students; the expectations of parents and other schools; underlying assumptions about learning and instruction.

The purpose of this section is to suggest some of the thinking which has gone into the curriculum decisions already made and provide a framework for those still to be made, both in general curriculum planning and in the daily decision-making of the classroom teacher. The treatment is necessarily brief. The best that can be done here is to "lay out the territory" and indicate other references where ideas can be explored in depth.

Those who are interested primarily in the substance of the program, and less in its rationale, may wish to skip this section entirely or come back to it later.

CONTENT AND GOALS OF THE MATHEMATICS PROGRAM

Considerations

Curriculum reform. The curriculum reform movement of the sixties brought to elementary school mathematics new vocabulary, new topics, new methodology, new materials, new goals, and re-ordered all this under the banner of "New Math." The abandonment of the term "arithmetic" in favor of "mathematics," with its broader scope is somewhat symbolic and reflective of the degree of change involved. Today there is a tendency to eliminate some of the more radical innovations, but many changes seem to have become firmly established in the curriculum, especially the integration of topics formerly considered as enrichment: geometry, graphing, statistics, probability, logical/mathematical reasoning, sets, etc.

Changing society. American society is changing rapidly. In a "Position Paper on Basic Skills" the National Council of Supervisors of Mathematics (1977) cites "the changing needs of society, the explosion of the amount of quantitative data, and the availability of computers and calculators" (p. 21) as reasons to re-examine ideas about the mathematical skills required for life in modern society. As part of the position paper, a list of "Ten Basic Skill Areas" (p. 20) was prepared; the list is reproduced here on page 16.

Back-to-basics. At the same time, a strong "Back-to-Basics" movement exists in the country, and more and more parents express dissatisfaction with their children's education. Related to this is a move toward minimal competency testing. These factors produce pressure to give greater emphasis to computational skills.

Parent expectations. When parents are asked about the goals they have for their children in math, they frequently cite: enjoyment of math, computational skill, knowledge of basic facts, being ready for _____ (next year, junior high, etc.)

Articulation with junior high school. Campus School students go on to a number of different junior high schools, and ability in mathematics often has important consequences for placement in junior high programs. There are widespread expectations that by the end of the elementary years, a child should have mastered a number of skills and topics: reading and writing numbers, basic number facts (tables), all whole number operations (addition, subtraction, multiplication, division), addition and subtraction of fractions (both common and decimal forms). More able students are further expected to have considerable skill with, or at the very least a solid introduction to, multiplication and division with fractions, decimals, percent, ratio and proportion. Speedy, accurate computation may be highly valued.

Impact on the Campus School Mathematics Program

Broadened content. The content of the program has been extended to include Geometry, Relations and Functions, Statistics, Logical/Mathematical Reasoning.

Metrics. Measurement with metric units is being systematically introduced.

Computational skills. The development of computational skills is given careful attention in the program. Provision has been made for developing speed and accuracy, particularly at the ages of 10- and 11-years when the child is preparing for the transition to junior high school.

Other skills. The goals and objectives of the program include such skills as estimating, judging reasonableness of results, organizing and interpreting data, recognizing and making practical applications, etc.

Attitudes and appreciations. The program recognizes in its goals and objectives the need for students to develop confidence in their ability to learn and master mathematics and to appreciate both the beauty and utility of mathematical ideas and skills, both in school and in daily life.

PAGE 16 "Ten Basic Skill Areas" REMOVED PRIOR TO BEING SHIPPED TO EDRS FOR FILMING DUE TO COPYRIGHT RESTRICTIONS.

ORGANIZATION AND DEVELOPMENT OF THE PROGRAM

Considerations

Cognitive development of children. Over the course of the years that children spend in the Campus School, from ages 3 to 12 years, major transformations take place in the cognitive development of the child. According to Piaget's description of cognitive development, these years are likely to include:

- Pre-operational stage (from about 1-1/2 to about 4 or 5 years old)
- Transition from pre-operational stage to concrete operational stage (from about 4 or 5 to about 7 or 8 years old)
- Concrete operations stage (from about 7 or 8 to about 11 or 12 years old)
- Transition to formal operations stage (from about 11 or 12 to about 14 years old)

(Isaacs 1974, pp. 22-24)

These years include two "critical periods in the intellectual life of the child" (Piaget 1964, p. 74): the transition to concrete operations, by around 7 or 8 years, and the transition to formal operations, beginning around age 11.

These changes do not occur at the same time for all children, and take place over a considerable period of time. The child's developing conceptions of number, logic, space, and measurement have been summarized in time-line format in figure 2. In the figure, each item is keyed to a specific outside reference so that the supporting information is easily accessible. The stage of the child's cognitive development clearly affects the kind of reasoning and understanding which is possible. For example, it is not until about age six that most children "get some first inklings of the true meaning of distance, length, number and the rest" (Isaacs 1974, p. 23), and Copeland (1974a) finds "a stage of unreadiness for addition in many children of six to seven years of age. These children are learning addition without benefit of the necessary thought processes" (p. 24) and later notes that children "may be beginning addition too soon and multiplication too late" (p. 27).

■ For an overview of Piaget's description of cognitive development, Isaacs (1974), A Brief Introduction to Piaget is highly readable.

■ For the implications for mathematics curriculum and instruction, the work of Copeland (1978), How Children Learn Mathematics, 3rd edition, is recommended. It combines two books formerly published separately, How Children Learn Mathematics--Teaching Implications of Piaget's Research, 2nd edition (1974b) and Diagnostic and Learning Activities in Mathematics for Children (1974a).

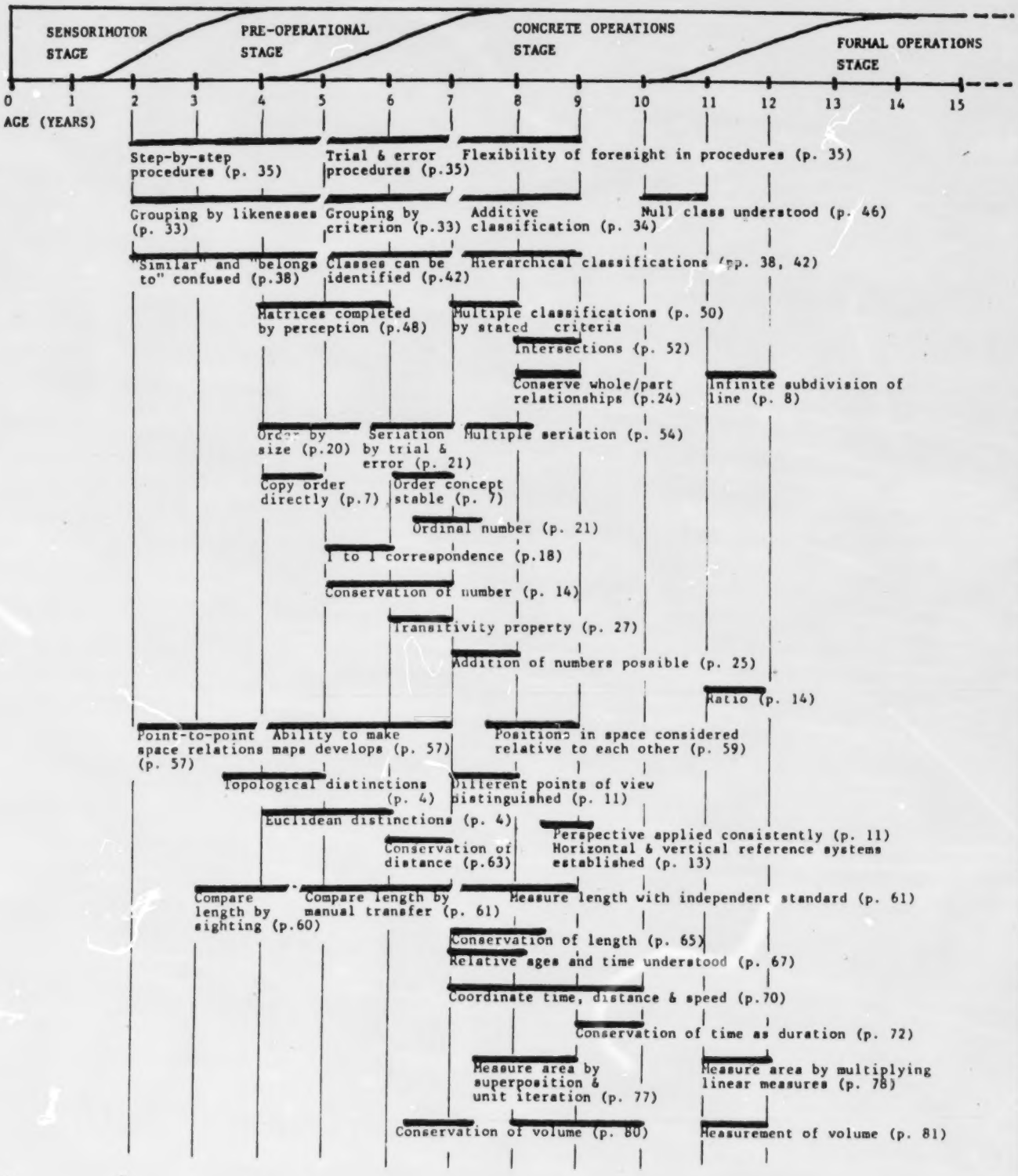
Impact on the Campus School Mathematics Program

Inclusion of logical/mathematical reasoning. While the development of the ability to reason logically is a concern in all areas of the school program, it has been specifically included in the mathematics program and identified in the list of strands.

Differentiation of program emphasis by levels. An attempt has been made to define the emphasis of the program for each level in a way that is compatible with the child's likely stage of cognitive development. The emphasis for each level and the contribution of each level to the total program is provided in the "Summary" on page 8. This differentiation is Premise 1 of the program (see page 4).

Placement and order of topics. Some adjustments have been made in the placement and order of topics in the program based on the implications of cognitive development. Work with subtraction as a computational skill has been delayed until the 8-9 level; multiplication is being introduced in the 6-7 level (see Copeland 1974a, pp. 24, 27). Fractional numbers are being introduced at the 8-9 level, and computational skill with fractions is being delayed until 10-11 level. Most work with relations and functions, including topics like ratio, percent, probability, is introduced at the 10-11 level. Instruction on the structural properties of the number system and the properties of operations is delayed until the 10-11 level. Topics in geometry at all levels below 10-11 de-emphasize formal Euclidean concepts.

Fig. 2. A TIME-LINE OF COGNITIVE DEVELOPMENT*


SENSORIMOTOR STAGE
PRE-OPERATIONAL STAGE
CONCRETE OPERATIONS STAGE
FORMAL OPERATIONS STAGE
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
AGE (YEARS)
Step-by-step procedures (p. 35)
Trial & error procedures (p.35)
Flexibility of foresight in procedures (p. 35)
Grouping by likenesses (p. 33)
Grouping by criterion (p.33)
Additive classification (p. 34)
Null class understood (p. 46)
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Relative ages and time understood (p. 67)
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Measure area by superposition & unit iteration (p. 77)
Measure area by multiplying linear measures (p. 78)
Conservation of volume (p. 80)
Measurement of volume (p. 81)

*Chart developed for this curriculum by Paul Williamson, based on Nathan Isaacs, A Brief Introduction to Piaget (New York: Schocken Books, 1974), pp. 22-24; and Richard W. Copeland, Diagnostic and Learning Activities in Mathematics for Children (New York: Macmillan Publishing Co., Inc., 1974).

Numbers in parentheses after specific items refer to pages in Copeland. The horizontal bar for each item indicates the approximate age range in which most children would be expected to perform consistently. Some children may develop a specific ability somewhat before that indicated, some after.

NATURE OF THE EDUCATIONAL EXPERIENCES

Considerations

Types of mathematical knowledge. There are different types of mathematical knowledge. In one scheme, four types are identified: concepts, generalizations, procedures, and number facts (Davis 1978). Frequently, the following scheme is used: concepts, skills, applications. An important idea about the different types of mathematical knowledge is that the learning of each type facilitates learning of the others. This interdependence is shown graphically in Figure 3.

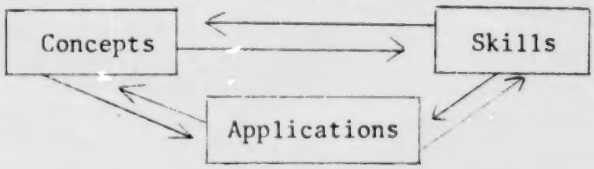

Fig. 3 THE INTERACTION OF CONCEPTS, SKILLS, AND APPLICATIONS
(Trafton 1974, p. 34)

■ For a discussion of these interdependent relationships, see Trafton (1974, pp. 32-34), "The Curriculum," and Hamrick and McKillip (1978, pp. 1-12), "How Computational Skills Contribute to the Meaningful Learning of Arithmetic."

■ For a discussion of the types of mathematical knowledge, see Davis (1978), "A Model for Understanding Understanding in Mathematics."

View of the learning process. There is no unified, universally-accepted theory of learning. Rather there are theories of learning, and teachers and curriculum planners are "faced with the necessity of identifying, understanding, and assessing the various theories of learning generated by research and psychologists, and selecting from these the components that best serve our curricular purposes" (Zais 1976, p. 245).

One category of learning theories can be called associationist. Associationist theory ". . . has probably exerted a greater influence on education and curriculum than any other school of psychological thought," and "has had overwhelming acceptance among learning theorists in the United States" (Zais 1976, p. 253). Generally, associationists view learning as "a process of building simple units into complex wholes; it is analogous to constructing a brick building" (Zais 1976, p. 253). This theory is "based on the assumption that learning involves formation of 'associative bonds' between stimulus and response events," and is most reasonable when dealing with basic skills that are "clearly definable and measurable" (Royer and Allan 1978, pp. 73-74). This conception leads "to the associationist practice of dividing curriculum content into its simplest components and 'programming' it so that the learner gradually acquires the simple units making up complex behaviors" (Zais 1976, p. 259).

An alternative to associationist theories is cognitive learning theory. In cognitive learning theories, the learner is not "merely a passive recorder of external events. Instead, the learner is viewed as a very active participant in the learning process" (Royer and Allan 1978, p. 120).

Field theorists (drawn from Zais 1976) view learning as "the sudden

restructuring of integrated wholes--in other words, insight" (p. 254). Perception is emphasized in field theories and is understood as perceptions of wholes. Learning occurs "when the organism has experienced a restructuring of perception of the total system (p. 259). In curriculum the field theorist "begins with major themes, concepts, ideas, and problems. He attempts to illuminate the structure of these wholes so that by a leap of insight the learner grasps the whole and its organizational principles" (p. 259). The "joy of discovery" accompanying such insight serves as intrinsic motivation for learning. The discovery method, "popularized among curriculum workers in the 1960s is rooted in this field-theory approach to learning" (p. 259).

Piaget in particular has concerned himself with the process by which the individual achieves cognitive restructuring. Whereas the field theorists held that "the principles of organization were innate" (Elkind 1976, p. 35), Piaget holds that

> . . . the starting point and crux of the child's intellectual growth is not--as it was long the fashion to assume--sensory perception or anything else passively impressed on him from outside, but his own action. (Isaacs 1974, p. 67)

The essence of Piaget's theory of learning seems to be that knowing is a result of the creative efforts of the individual:

> . . . the child constructs reality out of his experiences with the environment in much the same way that an artist paints a picture out of his immediate impressions. A painting is never a simple copy of the artist's perceptions . . .. A painting is always a unique combination of what the artist has taken from experience and what she has added to it from her own scheme of the world.
>
> In the child's construction of reality the same holds true. What the child understands reality to be is never simply a copy of what he has received by his sense impressions; it is always transformed by the child's own ways of knowing . . .. From Piaget's standpoint, we can never really know the environment, but only our reconstructions of it. Reality is always a reconstruction of the environment and never a copy of it. (Elkind 1976, p. 59)

Forman (1978) has summarized the process of equilibration by which Piaget believes cognitive restructuring takes place:

- Disturbance--"a conflict between something expected and something witnessed": "a sense that something is not quite right" (p. 6)
- Regulation--"actions or thoughts engaged to reduce the disturbance" (p. 6)
- Compensation--a way of thinking that "explains the surprising event that causes a disturbance" (p. 7)

The application of constructionist learning theory to curriculum would therefore center on creating disturbances which would trigger off the equilibration process. This, in turn, implies a prior understanding of the child's cognitive structures and a clear notion of how they might be changed. It should be clear, then, that devising curriculum and instruction informed

by constructionist learning theory demands a high degree of knowledge and clinical skill on the part of the teacher, and meticulous attention to design of the environment and the learner's experiences within it.

Jerome Bruner (1966) has addressed the question of how people process information and represent it. Bruner describes "three parallel systems for processing information and for representing it--one through manipulation and action, one through perceptual organization and imagery, and one through symbolic apparatus" (p. 28). These three modalities are often referred to as "enactive," "iconic," and "symbolic."

A key idea is that these modalities are parallel. It has long been taken as conventional wisdom, buttressed by the work of Piaget, that young children "need" materials. But it has also been taken as given that this need is a stage which passes, and that the trend and goal is to move on to the more sophisticated and abstract symbolic level, much as one moves from size 5 shoes to size 12, never to return again. One sees evidence of this assumption in the question, "Why are the 10-year olds still using Cuisenaire rods?" and in elementary classrooms barren of materials except for the textbook and a token bulletin board.

Bruner's contribution is to point out that learners continue to make use of all three modalities, that power in learning is related to flexibility in their use. Thus, Bruner hypothesizes that an optimum learning experience would include all three modalities and would likely progress from enactive to iconic to symbolic representation (p. 49). But what of the child who "doesn't need" to use concrete materials?

> When the learner has a well-developed symbolic system it may be possible to by-pass the first two stages. But one does so with the risk that the learner may not possess the imagery to fall back on when his symbolic transformations fail to achieve a goal in problem solving (p. 49).

By a similar argument, one can criticize instruction which relies solely on manipulative materials and seldom if ever teaches toward the symbolic.

Of course, some learning remains largely enactive, like walking:

> A centipede was happy quite,
> Until a frog in fun
> Said, "Pray, which leg comes after which?"
> This raised her mind to such a pitch,
> She lay distracted in the ditch
> Considering how to run.
> --Anonymous

The notions of concrete, iconic (semi-abstract), and abstract representation have gained widespread currency in curriculum and instructional theory (as for example in the conventional wisdom of going "from the concrete to the abstract"). These categorizations are even being explicitly used in some commercially available curriculum materials, such as Chip Trading published by Scott Resources. Bruner's theory of representation is part of the rationale for the extensive use of physical materials, models, etc. which characterize recent curriculum materials.

The curriculum practices associated with the associationist and with the cognitive-constructionist theories may be summarized as follows:

> Traditional education is based on a copy theory of knowing and assumes that if the child is given the words he will acquire the ideas that they represent. A constructionist [Piagetian] theory of knowing asserts just the reverse, that the child must attain the concepts before the words have meaning. Thus Piaget stresses that the child must be active in learning, that he must have concrete experiences from which to construct reality, and that only in consequence of his mental operations on the environment will he have the concepts that will give meaning to the words he hears and reads. (Elkind 1976, p. 60)

■ For a summary of associationist and field theories, see Zais (1976).

■ For further discussion of concrete, iconic, and abstract representation, see Bruner (1966).

■ For discussion of operant, associative and cognitive learning theories, and their application to teaching, see Royer and Allan (1978).

Characteristics of Campus School students. Classes at the Campus School generally include a range of ages and a wide range of achievement and aptitude in mathematics. However, it is usual to find most children to be of average or above-average ability with relatively few encountering severe difficulty. Most children get high scores on standardized achievement tests.

Campus School children are frequently described as being highly verbal. Indeed, they generally have well-developed language skills, and most learn to read early and well. This is an advantage, for a rich language base is essential for meaningful development of special mathematical vocabulary, language and symbols. However, high verbal proficiency may mask inadequate conceptual development, with the result that caution must be exercised in assessing children's understandings and skills. Well-developed reading skills facilitate the use of independent, individual or small-group activities.

Impact on the Campus School Mathematics Program.

Kinds of mathematical knowledge. The goals and objectives of the program include different types of mathematical knowledge: concepts, procedures, generalizations, number facts (or concepts, skills, applications). However, the emphasis upon these types of knowledge varies according to the age and abilities of the students.

Types of learning activities. The different kinds of learning included in the program call for different types of learning experiences. For example, the learning of skills and number facts are tasks compatible with associationist learning theories. On the other hand, learning to understand a procedure or a certain relationship (like the inverse relationship of multiplication and division) is a task involving new or modified cognitive structures, more compatible with a cognitive-constructionist view of learning. The concern for suitable learning experiences is reflected in Premise 2, p. 4.

Types of student behaviors specified. The goals and objectives of the program call for various ways to demonstrate learning and understanding. This may be noted in the use of terms and phrases like "construct models for . . . and solve," "recognize problem situations . . .," "relate . . . to . . . ." This reflects an intention to teach toward higher level cognitive skills and to ensure meaning for language and symbols.

Problems and Issues

A number of problems and issues relevant to instructional planning are indicated below.

Using structural properties to teach algorithms. In the "New Math" movement, extensive use was made of the structural properties of the number system to teach and justify algorithms. According to Trafton (1975) "although this use of structure had wide support initially, doubts were eventually raised about the appropriateness of such deductive approaches. Some felt that this approach was too formal and involved too much symbolization" (p. 24). This is in accord with Piaget's findings that formal deductive thought does not develop until adolescence. A simple example may be the use of expanded notation in the initial development of algorithms, such as for addition:

$$\begin{array}{r} 20 + 8 \\ +\ 40 + 7 \\ \hline 60 + 15 \end{array} = 60 + (10 + 5)$$
$$= (60 + 10) + 5$$
$$= 75$$

It is not until 8-9 years that the ability to conserve the whole-part relationship is usually established, with the result that for many children, exercises like the above have little meaning.

Equality concept. A recent investigation into children's understanding of the equality concept and the = sign found that

> . . . children consider the symbol = as a 'do something signal.' There is a strong tendency among all of the children to view the = symbol as being acceptable in a sentence only when one (or more) operation signs (+, -, etc.) precede it . . .. Moreover, it is important to observe that there is no evidence to suggest that children change in their thinking about equality as they get older and progress to upper grades . . .." (Behr, et al., 1976, p. 10)

Mathematics vocabulary. A particular and specialized vocabulary is associated with mathematics.

Mastery of vocabulary is certainly an aspect of mathematical literacy. But the question always arises of which terms to introduce, and how many of them should children be expected to remember. There is certainly no hard and fast rule that can be cited, but it may be helpful to discuss some ways to think about this question.

In some cases, a term may differ from general usage, and thus pose a

problem, as for example "set" or "average." In other cases, common usage can be helpful in comprehending a term, for example "commutative" (we commute to work) and "distributive" (we distribute books, pencils, etc.). Teachers should be sensitive to and capitalize on the mathematical knowledge and understandings embedded in the everyday language of the child.

Some terms are so general or occur so frequently that children should remember them and use them precisely: add, subtract, multiply, divide, digit, sum, product, etc. For example, it is inexcusable to refer to a problem like 23 x 35 as "2 numbers times 2 numbers" (misusing "number") when "digit" is available, or to say "pluses" instead of "addition."

Some terms are so specialized or occur so seldom that they may be considered of short-term utility. For example, in studying division, it is useful to have the terms "divisor" and "dividend" to refer to parts of the problem, but "dividend," particularly, has limited use at other times. Children should certainly be exposed to vocabulary of this type and learn to use it when necessary, but should not be expected to recall it over the long run. Instead, they should learn to recognize these terms and to refer to specialized mathematics glossaries for exact meanings when necessary.

Other terms, like "minuend" or "abscissa" may be obsolete or so unfamiliar or difficult that they are not worth even temporary usage, and should be ignored.

Use of manipulative materials. The use of manipulatives in math instruction has been the subject of considerable investigation. Findings are frequently mixed, but some conclusions seem clear:

° Some manipulatives are better for some purposes than others.

° Great care must be exercised to develop the relationship between the physical materials and the ideas they represent.

(Gerling and Wood 1976)

Trafton (1975) cites four purposes for manipulative materials:

(1) Exploratory experiences where the purpose is to provide readiness for later work.

(2) Development of a specific mathematical idea.

(3) Provide added experience with an idea that has already been developed.

(4) Source of experiences in which to extend or apply knowledge.

(pp. 38-39)

Use of class time. In their survey of the research literature on this question, Callahan and Glennon (1975) found that:

> There was general agreement among the studies that the classes which devoted 50 percent or more time to developmental activities performed better on achievement tests than those classes devoting 50 percent or more class time to drill and practice work. This was generally true for all ability levels of students, but increased time

on developmental activities may be most beneficial for the students who are mathematically talented.

Activities classified as "developmental" in these studies referred to classroom procedures intended to increase understanding of the number system, fundamental operations, and applications of number in everyday experiences. Activities such as teacher demonstrations, teacher explanations, group discussions, work with manipulative materials, and laboratory activities were classified as "developmental." In general, such individual pupil tasks as assigned exercises from textbooks, kits, dittos, and tapes were classified as "drill and practice" activities . . ..

Evidence would suggest that at least 50 to 75 percent of math class time should be devoted to meaningful developmental activities. Drill and practice activities should not be ignored, but 25 to 50 percent of class time appears to be ample. There is evidence that a disproportionate amount of class time is still spent on correcting homework and drill and practice activities. (pp. 95-96)

THREE- AND FOUR-YEAR OLDS

Overview

The curriculum at this level emphasizes background experiences and explorations with materials and their properties from which to develop concepts of number, space, and measurement. Emphasis is also placed on activities which stimulate the development of logical/mathematical reasoning.

In general, instruction is informal and based on the child's self-selected interaction with materials and activities offered.

Goals

The curriculum for three- and four-year olds is intended to enable the child to:

- Develop an awareness of number.
- Associate numbers and mathematical language with daily activities.
- Compare, sort, and order objects using attributes of the objects (length, color, shape, size, function, etc.).
- Develop concepts and vocabulary to describe spatial relationships (inside, outside, above, below, etc.).

Objectives

Logical/Mathematical Reasoning

° Sort a set of objects and state how each group formed is alike.

[° Classify a group of objects on the basis of a single property.]

° Complete a matrix (2 x 2) with one unknown.

° Order a group of objects (up to 5 [10]) according to a single property: color, length, height, size, etc. (→)

Number and Numeration: Whole Numbers

° Count and state the number of objects in a set (up to 10 [20]).

° Recognize and put in order the numerals 1 to 5 [10].

° Match the number of objects in a set with the numeral (1 to 5 [10]) and vice-versa.

° Compare the number of objects in 2 sets, by matching [by counting]: fewer, more, same number as.

Operations and Properties: Whole Numbers

Numbers and Numeration: Fractional Numbers

Operations and Properties: Fractional Numbers

Statistics

Relations and Functions

Geometry

(See also: Logical/Mathematical Reasoning)

° Recognize and identify examples of common shapes: circle, square, triangle, rectangle, oval, diamond, etc.

° Describe properties of shapes: curved, straight, zig-zag, corners, sides, etc.

° Describe spatial relationship of 2 objects: inside, outside; nearer, farther; between; over, under, on; first, next, last; [left, right], etc.

° Copy [and repeat] a pattern of color, shape, objects, etc.

[ ] Items for advanced students. (→) Continues in next level.

Measurement

(See also: Logical/Mathematical Reasoning)

- Compare directly objects according to:

  Length--longer, shorter, longest, shortest, same length

  Height--taller, shorter, tallest, shortest

  Weight--heavier, lighter

  Capacity (volume)--more, less, empty, full

  Size--bigger, smaller, biggest, smallest

FIVE-YEAR OLDS

Overview

The curriculum at this level emphasizes developing an understanding of number and the recognition and meaning of the numerals 1...20. Emphasis is also placed on experiences with measurement and geometry which extend the child's ability to discriminate and describe properties of objects and space and to order and classify logically on the basis of these properties.

In general, instruction at this level relies on a combination of guided exploration with structured materials and informal, teacher-directed activities.

Goals

- Order or classify objects on the basis of 1 [2] attributes.
- Understand the concept of number and state the number of objects in a set, up to 20 [30].
- Write numerals to 20 [30] using standard formation of digits 0...9.
- Identify common plane and solid shapes, both in isolation and in the environment.
- Compare objects on the basis of physical attributes: length, area, capacity (volume), weight.
- Associate numbers and mathematical language with daily activities.
- Develop an awareness of ability to solve mathematical problems and of the problem-solving process.

[ ] Items for advanced students.

Objectives

Logical/Mathematical Reasoning

(See also: Number and Numeration: Whole Numbers, Geometry, Measurement)

- Order a set of objects on the basis of a single attribute and state the basis of ordering. (CONT.) (→)
- Order sets of objects according to number.
- Match objects in 2 ordered sets, using their positions in the series.
- Copy and repeat a pattern of colors, shapes, objects, number, etc. [or of a combination of attributes, e.g., red cylinder, blue square ...].
- Classify a set of objects, using a single attribute, and state the basis of classification.

[° Classify a set of objects using 2 attributes, given a suitable matrix or overlapping circles.]

[° Identify a subset of a given set of objects]

Number and Numeration: Whole Numbers

- Write the digits 0...9 using standard formation.
- Count by rote to 100.
- Recognize and write numerals 1...20 [30].
- Count the number of objects in sets up to 20 [30],
- Compare the number of objects in 2 sets (up to 10 [20]) by 1:1 correspondence, and relate the numbers: greater than, less than, same number as.
- Identify positions in an ordered series, using ordinal numbers, first... fifth.

[° Construct models for numbers 10-19 using the concept "group of 10" (beginning place value).]

Operations and Properties: Whole Numbers

Addition

- Construct a model for sums to 10, and solve.

[° Record additions.]

[ ] Items for advanced students. (CONT.) Continued from previous level.
(→) Continues in next level.

Subtraction

° Construct a model for differences from sums to 10, and solve.

[° Record subtraction.]

Number and Numeration: Fractional Numbers

[° Name halves, given a model (informal, oral).]

Operations and Properties: Fractional Numbers

Statistics

° Make a list.

° Record numerical data with tallies. (→)

° Construct and interpret pictograph.

° Construct and interpret block graph.

Relations and Functions

Geometry

(See also: Logical/Mathematical Reasoning)

° Recognize and name common plane figures, in isolation and in the environment.

[° Describe properties of plane figures.]

° Recognize and name common solid figures, in isolation and in the environment: ball, can, box, cube.

[° Describe properties of solid figures.]

° Describe spatial relationship of 2 objects: top, bottom, middle; left, right; etc. [and combinations of descriptions, e.g., bottom left.]

Measurement

(See also: Logical/Mathematical Reasoning)

Time

° State time of day, e.g., morning, afternoon.

° Relate past and future time to present, e.g., tomorrow, yesterday.

° Name days of week, and current day.

[ ] Items for advanced students. (→) Continues in next level.

Money

° Identify penny, nickel, dime, quarter [and state values in terms of penny].

Length

° Compare the length of 2 objects by direct comparison and describe the relationships: longer than, shorter than, as long as.

° Order a set of objects (up to 10) according to length.

[° Compare the length of 2 objects, using indirect comparison, e.g., string, unmarked measuring stick, other object.]

Weight

° Compare the weight of 2 objects [2 combinations of objects] and describe the relationship: weighs more than, weighs less than, weighs the same as. (→)

[° Order 3 objects according to weight.]

Volume

° Compare the capacity of 2 containers and describe the relationship: holds more than, holds less than, holds the same amount as.

[° Order 3 containers according to capacity.]

[ ] Items for advanced students. (→) Continues in next level.

SIX- AND SEVEN-YEAR OLDS

Overview

The curriculum at this level emphasizes the development of the concepts of number and space themselves, and of the models, language, and symbols to express these ideas. See Fig. 4. Logical classification--a consideration of how things are the same or different--is an important correlate of number work. Measurement skills (length) are introduced and the concepts of volume and area are explored.

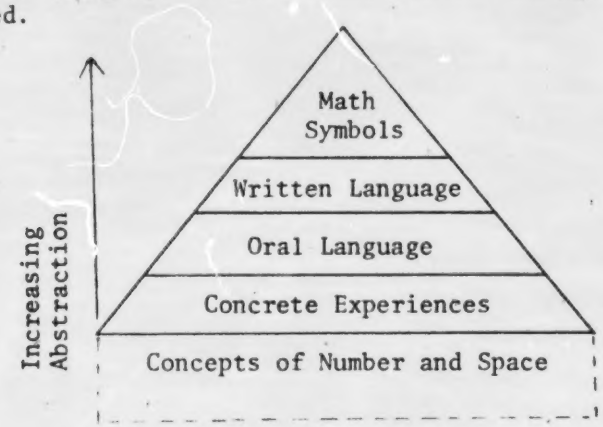

Fig. 4 REPRESENTING MATHEMATICAL IDEAS
(Adapted from Johnson, et al., 1977, p. 18)

In general, instruction at this level is a combination of guided exploration and teacher-directed group work. Physical materials are used extensively from which to draw mathematical ideas. Although some skills are isolated for practice, much of the work is approached through projects and activities which give meaning to the mathematics.

Goals

The curriculum for six- and seven-year olds is intended to enable the child to:

- Classify, order, or pattern objects according to 1 or more attributes.
- Read, write, and relate to models, numbers through 999.
- Understand the meaning of addition, subtraction, multiplication and construct models for any addition, subtraction, multiplication basic fact.
- Estimate and add 2- [3-] digit numbers with renaming, with and without models.
- Subtract 2- [3-] digit numbers with renaming with models.

[ ] Items for advanced students.

- Collect, organize, interpret numerical data using tables, charts, simple bar graphs.
- Develop concepts of area and volume. (See Nuffield, Shape and Size 2)
- Measure by direct comparison or by comparison with a reference unit (non-standard or standard metric).
- Use oral and written language and mathematical symbols to describe mathematical ideas.
- Identify personal progress in mathematics.
- Recognize and appreciate mathematics as a subject.

Objectives

Logical/Mathematical Reasoning

- Construct hierarchical or multiple classifications of objects using up to 2 attributes. (→)
- Order up to 10 objects using 1 attribute. (CONT.) (→)
- Recognize and construct patterns with objects or numbers. (→)

Numbers and Numeration

- Recognize, understand, and write using standard formation the following digits and symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, -, x, =, >, <.
- Read, write, and construct models for numbers to 999.
- Name place values for digits in units' period.
- Identify which number is greater or less, given numbers to 99.
- Count by 2's, 5's, 10's [3's, 4's, 100's].
- State ordinal number for positions 1 to 10 in a series.
- Recognize patterns of numbers.

Operations and Properties: Whole Numbers

Addition

- Construct models for concept of addition and for all basic addition facts; write an equation for the fact from the model.
- Recall or solve accurately (without models) all basic addition facts.
- Estimate and solve addition problems (to 2-digit addends with regrouping; 3 digits without regrouping), with or without models.
- Recognize and apply that addition is commutative.
- Recognize problem situations which require addition. (→)

Subtraction

- Construct models for concept of subtraction and for all basic subtraction facts; write an equation for the fact from the model.
- Recall or solve accurately (without models) at least 50% of basic combinations.

[ ] Items for advanced students. (CONT.) Continued from previous level.
(→) Continues in next level.

Subtraction (cont'd)

° Construct models for and solve subtraction problems (to 2-digits) with regrouping.

[° Estimate and solve subtraction problems (to 2- [3-] digits) without [with] regrouping, without models.]

[° Recognize that subtraction is not commutative.]

° Recognize problem situations which require subtraction. (→)

Multiplication

° Construct models for the concept of multiplication and for all basic multiplication facts; write equation for fact from the model.

° Recall and solve accurately the facts using 2, 5, or 10 as a factor.

° Construct a model for multiplication of 1-digit x 2-digits.

° Recognize that multiplication is commutative.

° Recognize problem situations that require multiplication. (→)

° Relate multiplication to addition. (→)

Division

[° Partition a set of objects ("sharing"), given a context, orally.]

Numbers and Numeration: Fractional Numbers

[° Name halves, thirds, fourths, given a model. (Informal, oral only)]

Operations and Properties: Fractional Numbers

Statistics

° Record data using tally marks. (CONT.)

° Organize data in chart or table form, including suitable headings and labels.

° Construct and interpret a bar graph from data in table form, including suitable headings and labels.

Relations and Functions

[ ] Items for advanced students. (CONT.) Continued from previous level.
(→) Continued in next level.

Geometry

(See also: Measurement)

° Estimate and measure surface by counting repeated units on the surface. (→)

° Identify shapes which tessellate and construct tessellations on a surface.

[° Construct plane figures with constant area and different shape.]

° Measure capacity (volume) by filling with unit solids or with liquids.

Measurement

(See also: Geometry)

Time

° Tell time from standard clock: hour, half-hour [15, 5 min.].

° State time from digital clock.

° Name calendar units for time [and tell how they are related]: day, week, month, year. (→)

° Read current day and date from calendar.

Money

° Tell value of coins: penny, nickel, dime, quarter.

° Count collection of coins, to $1.

Length

° Measure length in metric units (meter, centimeter [decimeter]) and state the measure as whole units or a combination of units, e.g., 1 m and 4 cm.

Weight

° Compare weights of 2 or more objects and order up to 3 objects by weight. (CONT.) (→)

Volume

° Measure volume of a container using non-standard units or liter.

° Compare volumes of 2 or more containers and order up to 3 [5] containers by volume. (CONT.)

Temperature

[ ] Items for advanced students. (CONT.) Continued from previous level.
(→) Continues in next level.

EIGHT- AND NINE-YEAR OLDS

Overview

The curriculum at this level emphasizes skills with operations on numbers and the development of remaining number facts, particularly those for subtraction and multiplication. For number facts the focus is on accurate recall of most facts and on strategies for quick solutions to those not recalled. Procedures for whole number computation are developed and extended to include standard algorithms for multiplication and division (by a 1-digit factor). Estimating and judging the reasonableness of answers are stressed, both in computation and measurement. Numeration is extended to reading, writing, and modeling larger numbers; fractional numbers are introduced. Concepts and skills are carefully developed and applied to real-world situations (see Fig. 5).

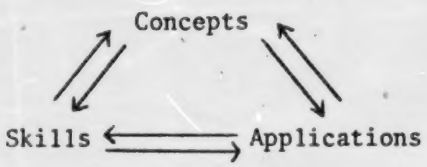

Fig. 5 INTERRELATIONSHIP OF CONCEPTS, SKILLS, AND APPLICATIONS (Trafton 1975, p. 34)

In general, instruction at this level is oriented toward development and practice with specific skills or topics but also includes many projects or activities in which skills are applied. Concepts and skills are carefully developed from and related to familiar models; extensive use is made of physical materials in addition to frequent use of textbooks.

Goals

The curriculum for eight- and nine-year olds is intended to enable children to:

- Classify and order objects and numbers (intersecting sets, matrices, subsets).
- Read, write, and understand whole numbers through thousands' period (to 999,999).
- Recognize and construct patterns, especially of numbers.
- Recall or solve quickly basic number facts for addition, subtraction, multiplication.
- Understand the meaning of arithmetic operations (addition, subtraction, multiplication, division) and recognize problem situations in which each is appropriate.

- Estimate and compute with whole numbers: addition (any), subtraction (any), multiplication (1-digit multiplier), division (1-digit divisor).
- Name fractional numbers, given models, as common fractions or in decimal form (.1).
- Collect, organize, and interpret numerical data using tables, graphs (bar, line).
- Recognize properties of geometric figures.
- Estimate and measure length, mass, volume, temperature using standard units. (Priority on metric units, but including customary units.)
- Apply concepts and skills to problem-solving.
- Identify personal progress in mathematics.

Objectives

Logical/Mathematical Reasoning

- Classify objects using intersecting sets, matrices, subsets (multiple or hierarchical classification). (CONT.)
- Order objects using single attribute. (CONT.)
- Recognize and construct patterns of objects or numbers. (CONT.)

Number and Numeration: Whole Numbers

- Read, write, construct models for numbers to 999,999.
- State place values and total values of digits through thousands period.
- Express numbers in expanded notation (total values of digits).
- Round numbers to largest place value.
- Identify special sets of numbers, e.g., square, cube, triangle, even, odd numbers.
- Identify factors or multiples of n.

Operations and Properties: Whole Numbers

Addition

- Recall all basic facts with speed and accuracy.
- Estimate and find sum of any 2 addends; of columns of 2- [3-] digit addends.
- Recognize problem situations when addition is required. (CONT.)

Subtraction

- Recall all basic facts with speed and accuracy.
- Estimate and find difference of any 2 numbers, with multiple regroupings.
- Solve subtraction problems stated as missing addend.
- Recognize problem situations when subtraction is required. (CONT.)

[ ] Items for advanced students. (CONT.) Continued from previous level.

Multiplication

- Construct model for concept and for any basic fact, e.g., using number line, array, repeated sets. (CONT.)
- Solve, without models, any basic fact, using appropriate thinking strategies.
- Construct model for and solve multiplication of 2- or 3-digit number x 1-digit multiplier.
- Estimate and compute, without models, multiplication of 2- or 3-digit number x 1-digit multiplier, using standard algorithm.
- Recognize problem situations when multiplication is required. (CONT.)
- Relate multiplication to addition. (CONT.)

Division

- Construct model for concept and for any basic fact.
- Construct models for and solve division by 1-digit factor.
- Estimate and compute, without models, division by 1-digit factor using standard algorithm.
- Recognize problem situations when division is required.

Number and Numeration: Fractional Numbers

- Name fractional parts, given models: geometric region, number line. (→)
- Name equivalent fractions, given models. (→)
- Read and write fractions (tenths [hundredths]), in decimal form, and relate to common fraction form. (→)

[° Name improper fractions and mixed numerals, given models.]

Operations and Properties: Fractional Numbers

Addition/Subtraction

- Add/subtract like common fractions, given model. (→)
- Add/subtract like decimal fractions, given model. (→)

[ ] Items for advanced students. (CONT.) Continued from previous level.
(→) Continues in next level.

Multiplication

[° Multiply whole number x fraction.]

Division

Statistics

(See also: Relations and Functions)

° Construct and interpret line graph or bar graph including appropriate labels, heading, etc.

° Record data in and interpret frequency tables.

° State range for set of data.

Relations and Functions

(See also: Statistics)

° Read, write, interpret ordered number pairs.

° Construct graph of ordered number pairs (First Quadrant only).

° Interpret scales on maps, drawings.

° Construct drawings or maps to scale.

Geometry

(See also: Measurement)

° Identify symmetrical figures and the lines of symmetry.

° Identify congruent figures by superposition.

° Recognize transformations of geometric configurations: flips, slides, rotations.

° Find perimeter of a figure in standard units.

° Find area of a figure by counting repeated units. (CONT.)

° Construct plane figures of different dimensions and constant area.

[ ] Items for advanced students. (CONT.) Continued from previous level.

Measurement

Time

- Tell time to nearest minute.
- State relationship of units for time, e.g., days/year, weeks/year, hours/day, etc. (CONT.)

Money

- Make change (amounts to $10.00).
- Recognize notation for money as decimal fractions.
- Compute with dollar and cents amounts (add, subtract, multiply, divide).

Length

- Name and recognize metric units for length, and state the relationships among them (cm, dm, m, [mm], [km]).
- Estimate and measure length to a given precision (e.g., nearer cm, dm, m).
- State measures as units and fractions of units, e.g., $3\frac{5}{10}$ [3.5] decimeters.
- Name, recognize, and measure using customary units for length; state the relations between units, e.g., 1 in. = 1/12 ft.; 1 ft. = 1/3 yd. (applied fractions).

Weight

- Compare weights of 2 or more objects and order objects according to weight. (CONT.)
- Measure weight in kilograms.

Volume

- Estimate and measure volume of containers in standard metric units (liter, milliliter).
- Estimate and measure volume of containers in customary units; state the relations between units, e.g., 1 qt. = 1/4 gal. (applied fractions).

Temperature

- Read a thermometer and record temperature: °C.

[ ] Items for advanced students. (CONT.) Continued from previous level.

TEN- AND ELEVEN-YEAR OLDS

Overview

The curriculum at this level emphasizes the extension and mastery of whole number computation with speed and accuracy and the development of concepts and skills with fractional numbers. Particular attention is focused on mathematical and logical reasoning and on understanding and applying generalizations about numbers and the way they work (see Figure 6).

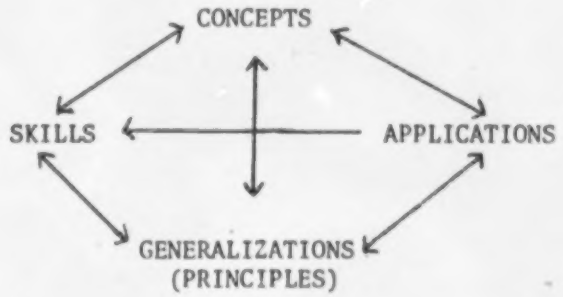

FIG. 6 RELATIONSHIP OF TYPES OF MATHEMATICAL KNOWLEDGE

Advanced students are introduced to topics such as ratio and proportion, probability, statistical techniques, etc.

In general, instruction at this level is oriented toward specific skills or mathematical generalizations. Integrative activities which call for the application of many skills and concepts are included. Physical materials (Cuisenaire rods, geoboards, chips, etc.) are used for the development of new ideas and skills, and for applications and extensions of old ones. There is an increasing reliance on the use of conventional written exercises, including the effective use of math textbooks. Regular homework assignments are often given. As part of the preparation for junior high school, students are also given experience with such techniques and practices as teacher lectures, formal tests, grading of assignments, etc.

Goals

The curriculum for ten- and eleven-year olds is intended to enable the child to:

- Develop awareness of reasoning logically and mathematically.
- Understand and apply the concepts of the place value system.
- Compute accurately with whole numbers.
- Recognize generalizations (properties) which apply to number operations and relationships among operations.

- Understand the concept of fractional numbers and perform appropriate computations.
- Describe and construct simple geometric figures, using geometry tools and instruments, e.g., protractors, compass, straightedge.
- Express mathematical ideas effectively orally and in writing.
- Apply concepts and skills to mathematical problems and topics (probability, statistics, measurement, etc.).
- Use textbooks and other instructional materials effectively.
- Identify personal progress in mathematics.

Objectives

Logical/Mathematical Reasoning

- Interpret statements: True/False, Negation, Inclusion/Exclusion (all/every, some, no/none).
- Distinguish between conclusions which are possible and those which are logically necessary.
- Construct diagrams to solve deductive propositions.
- Order items, given several relations among the items.

Number and Numeration: Whole Numbers

- Read and write numbers through billions, in standard and expanded forms.
- Understand concept of powers, and express powers with exponents.
- Understand and apply concepts of the place value system.
- Round numbers to a given precision.
- Read and write Roman numerals, and compare them with place value system.
- Identify factors (G.C.F.), multiples (and common multiples), [primes].

[° Name and recognize examples of negative numbers.]

Operations and properties: Whole Numbers

- Recall addition, subtraction, multiplication facts with speed and accuracy.
- Add any 2 whole numbers, or column of numbers.
- Subtract any 2 whole numbers.
- Multiply any whole number times 2- [3-] digit multiplier.
- Divide any whole number by 2- [3-] digit divisor.
- Identify inverse operations.
- Recognize properties for operations and their application to computational algorithms.

[ ] Items for advanced students.

Numbers and Numeration: Fractional Numbers

° Name and construct models for fractions. (CONT.)

° Rename fractions:

Improper to mixed numeral and vice versa
Proper fraction to higher or lower terms
Unlike fractions (mixed numerals) as like fractions (mixed numerals)

° Read and write fractions in decimal form, to .001.

° Recognize and apply identity property in renaming of fractions.

Operations and Properties: Fractional Numbers

° Add fractions, mixed numerals, decimals.

° Subtract fractions, mixed numerals, decimals.

° Construct models and multiply with whole numbers, fractions, decimals as factors.

° Construct models and divide with whole number, fractions, decimals as divisor or dividend.

° Recognize properties for operations and their application to computation with fractions.

Statistics

° Record, organize, interpret data in charts or frequency table. (CONT.)

° Construct and interpret graphs (bar, circle, line) from data, including use of headings, labels, scale if necessary, etc.

° Use sampling techniques to make estimations.

° Identify mean (average), median, mode, range for a set of data.

Relations and Functions

° Write ratios

° Write and solve proportions.

° Find percent of a number.

° Find number, given percent.

[° Identify probability that an event will occur.]

[ ] Items for advanced students. (CONT.) Continued from previous level.

Geometry

- Define properties of plane and solid figures, using concepts of angles, sides, faces, etc.
- Identify radius, diameter, circumference of a circle, and state their relationship.
- Measure and construct angles using protractor.
- Construct various geometric configurations using compass and straightedge: $\perp$, $\angle$, $\parallel$, hexagon, triangle, etc.
- Construct solid figures from patterns.

Measurement

- Compute with measurements (denominate numbers).
- Identify historical non-standard measures and describe need for standard measures.
- Understand and apply relationship among metric units.
- Select a suitable unit and measure length, volume, weight, temperature using metric units.
- Compute area and volume using square/cubic units.

EVALUATION

Preliminary evaluations have been done to ensure the quality of program planning and facilitate future curriculum planning and evaluation. These preliminary evaluations are described below.

Check for Internal Completeness and Consistency of Goals and Objectives

The goals and objectives of the program have been checked for internal completeness and consistency. Questions addressed in this evaluation are:

- Can each of the goals stated for the levels of the program be associated with one or more of the program goals?
- Are there program goals not reflected in the goals at any of the levels of the curriculum?
- Can each of the objectives for a level be associated with one or more of the goals for the level?
- Are there level goals which are not reflected in the objectives for the level?

To answer these questions, two matrices were constructed. The first relates the school goals with the goals of the various levels. The second relates the goals for each level with the objectives for the level.

■ The results of this evaluation are included in an appendix to the school's master copy of the Mathematics Program.

Instrument for Evaluation of Curriculum Planning

Curriculum planning for each level and classroom is expected to be a continuous process. To facilitate this process, simple instruments were devised to be used in evaluating the curriculum plans for each level or for the individual classrooms in the level. These instruments are matrices which relate the goals for a level with the learning experiences planned (or actually carried out). Questions which can be addressed with these instruments are:

- Can each of the learning experiences planned for the level (classroom) be associated with one or more of the goals?
- Are there goals not reflected adequately in the curriculum plans for the level (classroom)?
- Do the curriculum plans for the level (classroom) reflect the priorities and emphasis established for the level?

■ The matrices for the evaluation of curriculum planning are at the end of this section, pages 54-58.

Baseline Data on Achievement of Traditional Mathematics Objectives

The achievement of students in the school on traditional mathematics objectives is frequently of interest. The question addressed in this evaluation is:

° What is the achievement level of students in the school on traditional mathematics objectives?

The Campus School annually administers the Stanford Achievement Test to all pupils 8 years old and older. For purposes of this evaluation, this test was considered an adequate indicator of achievement of traditional objectives. National norms are provided for comparison with the wider population.

An analysis of the results of the Spring 1979 testing was done. This data can serve as a baseline for future evaluations and provide reassurance to parents and teachers.

■ The results of this evaluation are included in an appendix to the school's master copy of the Mathematics Program.

Further Evaluation

After the program has been implemented, an additional evaluation should be made to determine the extent to which the goals of the program are being achieved. The goals of the program are diverse, and many of them will be difficult to assess. Nevertheless, this evaluation should sample all the goals of the program. To do so it will be necessary to devise a variety of evaluation instruments. Any temptation to restrict the evaluation to those goals most easily measured must be strenuously resisted, for such restriction usually has the effect of narrowing the curriculum to those goals measured.

EVALUATION OF CURRICULUM PLANNING

The matrix shown on this page may be used in evaluating the curriculum plans for the level as a whole or for an individual classroom in the level. When completed, the matrix will show the relationship between the learning experiences planned and the goals of the level. It may be examined to answer questions like the following:

- Can each of the learning experiences planned for the level (classroom) be associated with one or more of the goals?
- Are there goals not reflected adequately in the curriculum plans for the level (classroom)?
- Do the curriculum plans for the level (classroom) reflect the priorities and emphasis established for the level?

Directions

(1) List the learning experiences planned across the top of the matrix. These may include "units," "activities," instructional practices (e.g. teacher-student conferences), or even aspects of the classroom (e.g. free-choice math center).

(2) For each learning experience, mark the goal(s) to which it relates. This may be done with a simple check or with a code like the following:

P = Primary focus of learning experience.

C = Contributes significantly to the goal, but is not a major focus of the experience.

LEARNING EXPERIENCES

GOALS																																
Develop an awareness of number.																																
Associate numbers and mathematical language with daily activities.																																
Compare, sort, and order objects using attributes of the objects (length, color, shape, size, function, etc.)																																
Develop concepts and vocabulary to describe spatial relationships (inside, outside, above, below, etc.)																																

(54)

FIVE YEAR OLDS

EVALUATION OF CURRICULUM PLANNING

The matrix shown on this page may be used in evaluating the curriculum plans for the level as a whole or for an individual classroom in the level. When completed, the matrix will show the relationship between the learning experiences planned and the goals of the level. It may be examined to answer questions like the following:

- Can each of the learning experiences planned for the level (classroom) be associated with one or more of the goals?
- Are there goals not reflected adequately in the curriculum plans for the level (classroom)?
- Do the curriculum plans for the level (classroom) reflect the priorities and emphasis established for the level?

Directions

(1) List the learning experiences planned across the top of the matrix. These may include "units," "activities," instructional practices (e.g. teacher-student conferences), or even aspects of the classroom (e.g. free-choice math center).

(2) For each learning experience, mark the goal(s) to which it relates. This may be done with a simple check or with a code like the following:

P = Primary focus of learning experience.

C = Contributes significantly to the goal, but is not a major focus of the experience.

LEARNING EXPERIENCES

| GOALS | | |

SIX AND SEVEN YEAR OLDS

EVALUATION OF CURRICULUM PLANNING

The matrix shown on this page may be used in evaluating the curriculum plans for the level as a whole or for an individual classroom in the level. When completed, the matrix will show the relationship between the learning experiences planned and the goals of the level. It may be examined to answer questions like the following:

- Can each of the learning experiences planned for the level (classroom) be associated with one or more of the goals?
- Are there goals not reflected adequately in the curriculum plans for the level (classroom)?
- Do the curriculum plans for the level (classroom) reflect the priorities and emphasis established for the level?

Directions

(1) List the learning experiences planned across the top of the matrix. These may include "units," "activities," instructional practices (e.g. teacher-student conferences), or even aspects of the classroom (e.g. free-choice math center).

(2) For each learning experience, mark the goal(s) to which it relates. This may be done with a simple check or with a code like the following:

P = Primary focus of learning experience.

C = Contributes significantly to the goal, but is not a major focus of the experience.

LEARNING EXPERIENCES

GOALS																															
Classify, order, or pattern objects according to 1 or more attributes.																															
Read, write, and relate to models, numbers through 999.																															
Understand the meaning of addition, subtraction, multiplication and construct models for any addition, subtraction, multiplication basic fact.																															
Estimate and add 2- [3-] digit numbers with renaming, with and without models.																															
Subtract 2- [3-] digit numbers with renaming with models.																															
Collect, organize, interpret numerical data using tables, charts, simple bar graphs.																															
Develop concepts of area and volume.																															
Measure by direct comparison or by comparison with a reference unit (non-standard or standard metric).																															
Use oral and written language and mathematical symbols to describe mathematical ideas.																															
Identify personal progress in mathematics.																															
Recognize and appreciate mathematics as a subject.																															

EIGHT AND NINE YEAR OLDS

EVALUATION OF CURRICULUM PLANNING

The matrix shown on this page may be used in evaluating the curriculum plans for the level as a whole or for an individual classroom in the level. When completed, the matrix will show the relationship between the learning experiences planned and the goals of the level. It may be examined to answer questions like the following:

- Can each of the learning experiences planned for the level (classroom) be associated with one or more of the goals?
- Are there goals not reflected adequately in the curriculum plans for the level (classroom)?
- Do the curriculum plans for the level (classroom) reflect the priorities and emphasis established for the level?

Directions

(1) List the learning experiences planned across the top of the matrix. These may include "units," "activities," instructional practices (e.g. teacher-student conferences), or even aspects of the classroom (e.g. free-choice math center).

(2) For each learning experience, mark the goal(s) to which it relates. This may be done with a simple check or with a code like the following:

P = Primary focus of learning experience

C = Contributes significantly to the goal, but is not a major focus of the experience.

GOALS	LEARNING EXPERIENCES
Classify and order objects and numbers (intersecting sets, matrices, subsets).	
Read, write, and understand whole numbers through thousands' period (to 999,999).	
Recognize and construct patterns, especially of numbers.	
Recall or solve quickly basic number facts for addition, subtraction, multiplication.	
Understand the meaning of arithmetic operations (addition, subtraction, multiplication, division) and recognize problem situations in which each is appropriate.	
Estimate and compute with whole numbers: addition (any), subtraction (any), multiplication (1-digit multiplier), division (1-digit divisor).	
Name fractional numbers, given models, as common fractions or in decimal form (.1).	
Collect, organize, and interpret numerical data using tables, graphs (bar, line).	
Recognize properties of geometric figures.	
Estimate and measure length, mass, volume, temperature using standard units (Priority on metric units).	
Apply concepts and skills to problem-solving.	
Identify personal progress in mathematics.	

TEN AND ELEVEN YEAR OLDS

EVALUATION OF CURRICULUM PLANNING

The matrix shown on this page may be used in evaluating the curriculum plans for the level as a whole or for an individual classroom in the level. When completed, the matrix will show the relationship between the learning experiences planned and the goals of the level. It may be examined to answer questions like the following:

- Can each of the learning experiences planned for the level (classroom) be associated with one or more of the goals?
- Are there goals not reflected adequately in the curriculum plans for the level (classroom)?
- Do the curriculum plans for the level (classroom) reflect the priorities and emphasis established for the level?

Directions

1) List the learning experiences planned across the top of the matrix. These may include "units," "activities," instructional practices (e.g. teacher-student conferences), or even aspects of the classroom (e.g. free-choice math center).

2) For each learning experience, mark the goal(s) to which it relates. This may be done with a simple check or with a code like the following:

P = Primary focus of learning experience

C = Contributes significantly to the goal, but is not a major focus of the experience.

LEARNING EXPERIENCES

| GOALS | | |

VII. REFERENCES

REFERENCES

Behr, Merlyn, Erlwanger, Stanley, and Nichols, Eugene. How Children View Equality Sentences. PMDC Technical Report No. 3. ERIC Document No. ED 144 802, 1976

Bruner, Jerome. On Knowing: Essays for the Left Hand. New York: Atheneum, 1970, c1962.

_____. Toward a Theory of Instruction. New York: W. W. Norton & Co., Inc., 1966.

Callahan, Leroy C. and Glennon, Vincent J., eds. Elementary School Mathematics: A Guide to Current Research, 4th ed. Washington, D.C.: Association for Supervision and Curriculum Development, 1975.

Copeland, Richard W. Diagnostic and Learning Activities in Mathematics for Children. New York: Macmillan Publishing Co., Inc., 1974a.

_____. How Children Learn Mathematics--Teaching Implications of Piaget's Research, 2nd ed. New York: Macmillan Publishing Co., Inc., 1974b.

_____. How Children Learn Mathematics, 3rd ed. New York: Macmillan Publishing Co., 1978.

This third edition includes the diagnostic and learning activities formerly published as a separate volume.

Davis, Edward J. "A Model for Understanding Understanding in Mathematics." Arithmetic Teacher v26 n1 (September 1978): 13-17.

Dewey, John. Democracy and Education. New York: The Free Press, 1930.

Elkind, David. Child Development and Education; a Piagetian Perspective. New York: Oxford University Press, 1976.

Forman, George. "Developmental Theory-Piaget" (manuscript, 1978).

Gerling, Max and Wood, Stewart. Literature Review: Research on the Use of Manipulatives. ERIC Document No. ED 144 811, 1976.

Hamrick, Katherine B. and McKillip, William D. "How Computational Skills Contribute to the Meaningful Learning of Arithmetic." In Developing Computational Skills, pp. 1-12. 1978 Yearbook, National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 1978.

Hawkins, David. "Messing About in Science." In Open Education: The Informal Classroom, pp. 58-70. Ed. by Charles H. Rathbone. New York: Citation Press, 1971. Originally published in Science and Children v2 n5 (February 1965).

Isaacs, Nathan. A Brief Introduction to Piaget. New York: Schocken Books, 1974.

Johnson, Judy, and others. Math in Oregon Schools. ERIC Document No. ED 148 621, 1977.

National Council of Supervisors of Mathematics. "Position Paper on Basic Skills." Arithmetic Teacher v25 n1 (October 1977):19-22.

Nuffield Mathematics Project. Shape and Size 2. New York: John Wiley & Sons, Inc., 1967.

Piaget, Jean. Judgment and Reasoning in the Child. Paterson, N.J.: Littlefield, Adams & Co., 1964.

Royer, James M., and Allan, Richard G. Psychology of Learning: Educational Applications. New York: John Wiley & Sons, Inc., Publishers, 1978.

Taba, Hilda. Curriculum Development: Theory and Practice. New York: Harcourt Brace Jovanovich, Inc., 1962.

Trafton, Paul. "The Curriculum." In Mathematics Learning in Early Childhood, pp. 16-41. National Council of Teachers of Mathematics, 37th Yearbook. Reston, VA: National Council of Teachers of Mathematics, 1975.

Zais, Robert S. Curriculum: Principles and Foundations. New York: Thomas Y. Crowell Company, 1976.